Wall Effects in Laminar Flow of Fluids through Packed Beds

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A model which accounts for the effect of a confining wall and the resulting voidage variations near it on the flow behavior in packed columns is proposed. The model, which is restricted to columns randomly packed with spheres of uniform size, reasonably describes the pressure drop-flow rate relationship for both linear and non-Newtonian fluids. The mass flux distribution is also studied in order to provide an insight into processes in which the flow behavior near the wall may be important.

SCOPE

Many of the existing studies on the flow of either Newtonian or non-Newtonian fluids through columns packed with spheres have been conducted with column-to-particle diameter ratios less than 30:1 and several have used ratios as low as 10:1. Frequently, there are compelling reasons of experimental convenience for the use of such large particles or small columns, especially in laboratory reactors employing packed beds, and even in commercial reactors the by-passing of the bulk of a bed due to rapid flow in the wall region may need to be assessed. While the possible importance of "wall effects" has long been known, there has been much confusion over the exact magnitude and radial distribution of flow irregularities as the wall of a packed bed is approached, especially in the analyses of pressure drop-flow rate relationships for non-linear fluids, for which these wall effects are much larger.

Several excellent experimental studies have measured, in the wall region, the non-random and periodic variations in porosity of beds packed with spheres of uniform size. These results are coupled with a model for the pore geometry to predict the attendant radial flux variations, and to provide thereby a new and important insight for the design and interpretation of experimental data for both linear and non-Newtonian fluids. The pitfalls in applying the traditional single region model are described, as is the need for further experimental and theoretical work in the area.

CONCLUSIONS AND SIGNIFICANCE

This model divides the bed into three regions, a wall region which extends a distance of one particle diameter away from the wall, a transition region in which appreciable voidage oscillations occur, and a bulk region in which the porosity is essentially uniform. For columns with a bed-to-particle diameter ratios less than about 30, it is found that the transition region occupies the largest fraction of the total flow cross section. It has been shown that the mass fluxes in the *transition* and *wall* regions are *higher* than in the bulk region by as much as 50% and 10% for fluids with power law indices of 0.25 and 1.0, respectively. This channeling effect occurs over a region of about 6 particle diameters in thickness (near the wall), and may have to be given adequate consideration in the design of packed columns, especially in cases where heat transfer or residence time considerations are critical, as in chemical reactors. However, more work is needed to quantify the details of the flow structure in this region.

This work reveals that the use of a single region model based on the average porosity in the bed leads to an over-prediction of the average mass flux. The triregional model proposed here appears to fit the available experimental data for beds with small D_c/D_p (column to particle diameter ratio) more accurately, in addition to revealing the flow distribution in the regions considered. If, however, wall effect corrections are to be avoided, packed columns with bed to particle diameter ratios greater than about 30 for Newtonian fluids, and about 50 for non-Newtonian fluids, should be used.

Finally, it is suggested that any correction that is made to account for the wall-effect, in predicting pressure drop-flow rate relationships, will be sensitive to the method of packing, particle shape and size distributions, and to the rheological properties of the fluid under consideration.

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BACKGROUND AND LITERATURE REVIEW

The pioneering works of Furnas (1929) and of Graton and Fraser (1935) have shown, through visual observations of the packing arrangement, that the porosity of the bed is greater in the region next to the wall. These workers concluded that the permeability of the bed, as defined by Darcy's Law:

$$k = \frac{GL\mu}{\Delta p\rho} = \frac{\mu_{ap}}{\rho}\frac{GL}{\Delta p} \tag{1}$$

in the wall region is greater than the bulk region. This effect, they ascertained, would lead to a larger apparent permeability of a packed bed, with decreasing value of the bed to particle diameter ratio, and Furnas (1929) determined this experimentally. Different results were obtained by Uchida and Fujita (1934), who found that in a set of three experiments, two cases where the column diameter had no effect, and one in which the permeability decreased with decreasing column to particle diameter ratio. Leva (1947) argued that if the wall effects lead to an increase in the porosity of the bed, the use of experimentally determined average porosity (i.e., from displacement experiments) should be sufficient and no additional correction is necessary.

Coulson (1935) and later, Carman (1937), Sullivan and Hertel (1940), and Coulson (1949) maintained that the only necessary correction for the effect of the wall is that for fluid friction on the surface of the wall. Their analysis can be written in equivalent form as:

$$\Delta p/L \;\alpha\; G/\phi_g \tag{2}$$

Where ϕ_g is an empirical correction term defined as:

$$\phi_g = \left[\frac{S_w}{S_w + C'S_c}\right]^{C''} \tag{3a}$$

S_w is the surface area per unit volume of the column, S_c is the packing surface area per unit volume of the empty column, and C' and C'' are empirical parameters. G is the average mass flux through the packed bed, and $\Delta p/L$ is the pressure drop per unit length of the column. The above researchers then postulated that the improvement in correlating of experimental results using the factor ϕ_g (which is always less than unity) supports their claim that the effect of the wall is in offering additional resistance and hence the pressure drop (for a given mass flux) for a column of infinite ratio of D_c/D_p (column to particle diameter ratio) is *less* than that for a column with a finite D_c/D_p ratio. This appears to be dramatically opposed to the views reported earlier in this section. In dealing with the "wall effect" quantitatively, Coulson (1935) has originally suggested a value of unity for the parameters C' and C'' in Eq. 1. Carman (1938) and Coulson (1949) have later accepted a value of 1/2 for C' and 2 for C'' to give the best improvement in correlating experimental results. The correction term ϕ_g (Eq. 3a) can be rewritten more conveniently as:

$$\phi_g = \left[\frac{1}{1 + \frac{2}{3}C'\frac{D_p}{D_c(1-\epsilon)}}\right]^2 \tag{3b}$$

For example, if C' is taken to be 1/2 and the bed porosity ϵ is 0.4, ϕ_g ranges from 0.81 at $D_c/D_p = 5$ to 0.96 at $D_c/D_p = 30$. The experimental work of Leva and Grummer (1947) has clearly demonstrated that the porosity of a packed bed does depend on the size of the column. Their results indicate that the bed porosity increases with decreasing D_c/D_p ratio for both random and non-random packing, with either uniform or mixed size spheres. Rose (1945) has suggested that the wall effect is primarily due to a lower porosity near the wall. His experimental results (Rose, 1945, 1948) were in partial agreement with the experimental results of Coulson (1935, 1949) and of Carman (1937) and they are in support of the general observation that for a given pressure drop the prediction of the mass flux, based on the average bed porosity, overestimates the actual mass flux; this deviation increases with decreasing D_c/D_p ratios.

The above pioneering studies have demonstrated the presence of a "wall-effect" for beds packed with spheres. The effect is partially due to the different arrangement of the spheres near the wall from that in the main body of the packing, and in addition, the surface of the wall may also offer some resistance to the flow. The implication of the above studies is that one should use large D_c/D_p ratios where possible in order to reduce the "wall-effect." Unfortunately, a large body of literature, both old and recent, exists, in which packed beds with small D_c/D_p ratios were used. It is therefore, imperatuve that a better understanding of the "wall effect" is obtained in order to evaluate experiments on the flow through packed beds more critically.

FLOW MODELLING

Choice of a Geometrical Pore Model

In modelling the flow of Newtonian fluids through packed beds, Blake (1922), Burke and Plummer (1928), Carman (1937), Kozeny (1927), Leva (1947), Coulson (1935, 1949), Ergun (1952), and MacDonald, El-Sayed, Mow and Dullien (1979) have shown that the use of the parallel channel model for the porous medium, in which the concept of the hydraulic radius is used, gave an adequate representation of the pressure drop-flow rate relationship. Many other models have been proposed to describe the complex geometry of the porous media (Dullien, 1975a, 1975b; Dullien and Azzam, 1972, 1973; Payatakes et al., 1973a, 1973b; Payatakes and Neira, 1977; Sheffield and Metzner, 1976; Sheidegger, 1974; Elata and Takserman, 1976; Fedkiw and Newman, 1979; Deiber and Schowalter, 1979). The use of such more sophisticated geometric models may be important, of course, in modelling additional or second-order effects, such as fluid trajectories and residence time distributions, but they seem to offer no measurable improvement in prediction of pressure drop-flow rate relationships. MacDonald et al. (1979) especially have demonstrated the excellence of performance of the capillary model over a wide range of conditions. Therefore, we will employ this most simplistic model. What we do in the following analysis can, of course, be extended to more complex models of pore geometry if the reader should wish to do so.

The Parallel Channel Model (Capillary Model)

Carman (1937) presented the parallel channel model with remarkable clarity, but unfortunately his interpretation has not been applied properly by all subsequent investigators (see Dullien, 1975a). The packed bed is modelled by a bundle of parallel channels of appropriate geometry. The flow through each of these channels can be assumed to follow the same expression as for a flow through a tube, but with an effective diameter replacing the tube diameter. In general, for both Newtonian and power law fluids, in the absence of inertial effects the velocity-pressure drop relationship becomes (Christopher and Middleman, 1965):

$$V = \left[\frac{4n}{3n+1}\right]\left[\frac{\Delta P}{2KL_e}\right]^{1/n}\frac{(D_e/2)^{n+1/n}}{2K_o} \tag{4}$$

Where V is the velocity through the channel (interstitial velocity), L_e is the length of the channel or alternatively, the effective path length followed by the fluid, and D_e is the effective channel diameter. The factor K_o essentially accounts for the inadequacy in the choice of a proper effective diameter, D_e. In the case of a round tube, the effective diameter is the true tube diameter, $K_o = 2$ and Eq. 4a reduces to the usual equation for the flow of power law fluids through a tube (Christopher and Middleman, 1965). The usual choice of an effective diameter is based on the concept of the hydraulic radius (Bird et al., 1960), where the effective diameter is replaced by four times the hydraulic radius. The hydraulic radius is defined as:

$$R_h = \text{volume for flow/wetted surface area} \tag{5}$$

hence, Eq. 4 becomes:

$$V = \left[\frac{4n}{3n+1}\right]\left[\frac{\Delta p}{2KL_e}\right]^{1/n}\frac{(2R_h)^{\frac{n+1}{n}}}{2K_o} \quad (6)$$

The value of K_o can in principle be calculated for a channel of a known geometry by comparing Eq. 6 with the exact solution. For example, for a flow through a slit $K_o = 3$. Unfortunately, in the case of porous media, the shape of the channels is rather complex; hence, there is no *a priori* knowledge of the value of K_o. Consequently, K_o must be determined experimentally. However, prior to that, one must replace the interstitial velocity by the superficial velocity which can be measured experimentally; in addition, L_e must be related to a measurable length scale, i.e., the length of the bed. From the consideration of continuity, the flow rate through any cross-section of the bed which is perpendicular to the mean flow is:

$$Q_n = V_n \epsilon A = V_o A \quad (7a)$$

where ϵ is the bed porosity and V_o is the superficial velocity. Consequently, the interstitial velocity component parallel to the mean flow is:

$$V_n = V_o/\epsilon \quad (7b)$$

Equation 7b is known as Dupuit's Assumption (Dupuit, 1863). Many investigators have used Eq. 7b to replace the velocity term in Eq. 6. This may be an oversimplification since V_n, the interstitial velocity component normal to the radial cross sectional area of the bed, is not equal to V in Eq. 6. The additional relation which is needed in order to eliminate V is obtained by realizing that the time of travel of a fluid element between two points is independent of the frame of reference used. Consequently, a distance L_e is covered in time:

$$t = L_e/V \quad (8a)$$

If the distance traveled in the axial direction of the packed bed for the path L_e is L, then:

$$t = L/V_n \quad (8b)$$

From Eqs. 8a and 8b,

$$\frac{L_e}{V} = \frac{L}{V_n} \quad (8c)$$

and by substituting in Eq. 7b

$$V = \frac{V_o}{\epsilon}\frac{L_e}{L} \quad (9)$$

Equation 6 can now be rewritten as:

$$V_o = \left[\frac{4n}{3n+1}\right]\left[\frac{\Delta P}{2KL}\right]^{1/n}\frac{(2R_h)^{\frac{n+1}{n}}}{2K_o}\frac{\epsilon}{(L_e/L)^{(n+1)/n}} \quad (10a)$$

Or, in terms of the mass flux through the bed

$$G = \rho\left[\frac{4n}{3n+1}\right]\left[\frac{\Delta P}{2KL}\right]^{1/n}\frac{(2R_h)^{\frac{n+1}{n}}}{2K_o}\frac{\epsilon}{(C^*)^{(n+1)/n}} \quad (10b)$$

where $C^* = L_e/L$.

Since there are two unknown parameters in Eq. 10b, they cannot be uniquely determined and are usually grouped together. For Newtonian fluids ($n = 1$) the two constants become:

$$K_o(C^*)^2 = B(n = 1) \quad (11a)$$

and are determined experimentally. For power law fluids B is a function of the power law index, hence

$$K_o(C^*)^{\frac{n+1}{n}} = B(n) \quad (11b)$$

This fact has been overlooked by many investigators, as has been recently pointed out by Kemblowski and Michniewicz (1979).

If K_o and C^* are universal constants, then a plot of $ln[B(n)]$ *vs* $n+1/n$ should yield a straight line with a slope of $ln(C^*)$. In this way, both K_o and C^* may be determined by "calibrating" a bed with both Newtonian and non-linear fluids. It is interesting to note that for a given packed bed, the parameter $B(n = 1)$ is remarkably constant, as is usually revealed by the linearity of the pressure drop flow rate relationships. This may suggest that K_o and L_e/L are unique parameters for a particular packed bed. However, a dependency of the value of $B(n = 1)$ on the porosity, particle shape and roughness, particle size distribution and the bed to particle diameter ratio (D_e/D_p) have been reported. Recently, MacDonald, Sayed, Mow and Dullien (1979) have shown through extensive data analysis that a better performance of the capillary model is obtained for $n = 1$ if $B(n = 1)$ is allowed to depend on the hydraulic radius, although in the viscous flow regime for a variety of packings a value of 5 for $B(n = 1)$ is acceptable. The same mean value for $B(n = 1)$ was reported many years earlier by Coulson (1935, 1949) and Carman (1937). For spherical particles, MacDonald et al. (1979) suggest at $B(n = 1)$ value of about 4.35. Morcom (1946) and Doljes (1978) have reported a mean value of 4.5 for spheres, while Mishara et al. (1977) and Brea et al. (1976) have suggested a value of 4.44. Bird et al. (1960) have suggested a value of 25/12 for L_e/L, which in view of Eq. 10b should be associated with $(L_e/L)^2$. They also adopted the capillary model, and hence $K_o = 2$, which leads to a value of 4.166 for $B(n = 1)$ as is also suggested by the work of Ergun (1952) with granular beds. On the other hand, the studies of James and McClaren (1975), Hanna et al. (1977), Bird (1965), Sadowski and Bird (1965), and Kemblowski and Michniewicz (1979) support a value of $B(n = 1) = 5$. From the above discussion, it is apparent that the value of $B(n = 1)$ for beds packed with spheres ranges from about 4.166 to 5.0. This means that the maximum scatter in most reported pressure drop-flow rate relationships for Newtonian fluids is about 20% (for beds with small D_e/D_p ratios the deviation is expected to be higher). For nonlinear fluids, the resulting error depends on the values assigned to K_o or L_e/L. For example, if we take $K_o = 2$, L_e/L ranges from 1.43 to 1.58, and hence the scatter (in using Eq. 10b) ranges from 20% for $n = 1$ to 64% for $n = 0.25$. It is not the intention of this study to establish the most likely value of $B(n)$, however, we do note that the available literature tends to support a value of $B(n = 1) = 5.0$ for beds of D_e/D_p ratio greater than about 30 packed with small spheres ($D_p \leq 0.1$ cm) of a narrow size distribution. The difficulty in establishing the "true" value of $B(n)$ from existing data on nonlinear fluids may be partially due to failing to consider the wall effect. Also, many of the existing studies on nonlinear fluids rarely determine the porosity and mean particle diameter independently. One usually determines either one of D_p or ϵ by an independent measurement, and the other via permeability measurements with Newtonian fluids. The uncertainty in the values of K_o and L_e/L and the additional complication of improper use of Eq. 10b suggest that the use of the capillary model for accurate predictions, especially for nonlinear fluids, warrants more systematic and careful work to establish the variations of K_o and L_e/L with the packed bed properties.

The Hydraulic Radius and the Wall Effect

The hydraulic radius (Eq. 5) for a bed composed of spherical particles can be shown (Bird et al., 1960) to be:

$$R_h = \frac{\epsilon D}{6(1-\epsilon)} \quad (12)$$

Mehta and Hawley (1969) have attempted to account for the wall effect by modifying the expression for the hydraulic radius. Their modification takes the form:

$$R_h = \frac{\dfrac{\text{Volume of voids}}{\text{Volume of bed}}}{\dfrac{\text{Wetted surface area of spheres}}{\text{Volume of bed}} + \dfrac{\text{wetted surface of wall}}{\text{Volume of bed}}} \quad (13)$$

which then reduces to:

$$R_h = \frac{\epsilon D}{6(1-\epsilon)M} \quad (14)$$

where

$$M = 1 + \frac{4D_p}{6D_c(1-\epsilon)} \quad (15)$$

It can be easily shown that the correction term ϕ_g (Eqs. 2 and 3) proposed by Carman (1937) and Coulson (1935, 1949) with $C' = 1$ is related to M through the following relation:

$$\phi_g \propto \frac{1}{M^2} \quad (16)$$

Consequently, the correction proposed by Mehta and Hawley (1969) is identical to that obtained by Carman, Coulson, and Sullivan and Hertel although it was derived from a different point of view. A similar approach was used by Dolejs (1978) for the flow of Newtonian fluids through packed beds. Although his approach is somewhat different in its derivation, the equivalent of a modified hydraulic radius can be defined as in Eq. 14 where:

$$M' = 1 + \frac{4}{9}\frac{D_p}{D_c(1-\epsilon)} \quad (17)$$

By setting $C' = 2/3$ in Eq. 3b, Eq. 16 results with M replaced by M'.

The approach of Mehta and Hawley was also adopted by Park et al. (1975) and by Hanna et al. (1977) in their studies of the flow of polymer solutions through packed beds.

All of the correction methods described so far consider what can be thought of as an effect hydraulic radius which is claimed to apply over the entire cross-section of the column. This is difficult to accept, since, physically, the effect of the wall should be confined to the "wall region" without affecting the nature of the hydraulic radius in the bulk of the porous bed.

The purpose of the remainder of this study is to present a simple model that will allow the evaluation of "wall effects" for both linear and non-linear fluids. For this purpose, the nature of the voidage variations near the wall of a packed column is first explored. Secondly, a model for the flow-rate-pressure drop relationship is proposed. The model retains the capillary model for the pore geometry, but takes into account the voidage variations in the packed column.

VOIDAGE VARIATIONS NEAR A SOLID WALL

Voidage variations near a solid wall in packed columns have been given support by measurements of radial velocity profiles. The early work of Saunders and Ford (1940) explored the radial velocity profile across the exit surface of a packed bed using a Pitot tube. Their results indicated that the velocity is approximately constant across the bed, except in a ring of about one particle diameter in width at the wall, in which the velocity was found to be about 50% greater than the bulk velocity. Schwartz and Smith (1953) have also studied the flow distribution in packed beds. Their results for beds with D_c/D_p less than 30 showed a peak velocity at about one particle diameter away from the wall; the velocity here ranged from 30 to 100% greater than the bulk velocity. Their results clearly demonstrated the effect of an increase in the porosity as the column wall is approached. A more recent study by Marivoet et al. (1974) has also confirmed the presence of voidage variations via measurements of velocity profiles.

A more accurate description of the nature of the porosity variations near the solid wall has been obtained by actual experimental measurements of the local porosity variations in beds packed with spheres (Benenati and Brosilow, 1962; Roblee et al., 1958; Ridgway and Tarbuck, 1967, 1968; Kubie, 1974; Haughey and Beveridge, 1966; Pillai, 1977). The technique employed by Benenati and Brosilow (1962) and Roblee et al. (1958), for example, consisted of pouring wax or epoxy resin into the bed to maintain the spheres in position for subsequent machining of radial increments. The porosity in each section was then calculated from a simple material balance. A different approach was taken by Ridgway and Tarbuck (1968), who developed a technique based on the rapid rotation of a cylindrical bed. The addition of known quantities of water which centrifuged to the outside of the bed, coupled with the determination of the thickness of the annular ring so formed, give a measure of the radial voidage variations in the cylindrical bed. Other methods are described in the work of Kubie (1974) and the papers of Haughey and Beveridge (1966) and Pillai (1977).

All of the above studies show that the presence of a confining wall introduces a source of *non-random* variations in the void fraction. The layer of spheres nearest to the wall tends to be highly ordered, with most of the spheres touching the wall. The subsequent layers build up in a progressively less ordered fashion until a fully randomized arrangement is achieved. The pioneering work of Roblee, Baird and Tierney (1958) with cylindrical beds packed with spheres, indicates a decrease in the void fraction from a value of unity at the wall to a minimum value of about 0.23 at a distance of about half a particle diameter from the wall. This is followed by an increase to a value of 0.55 at a distance of about one particle diameter from the wall. Beyond a distance of 1 particle diameter, the porosity oscillates in a damped fashion up to a distance of about 5 particle diameters beyond which the porosity is effectively constant. Similar results were obtained by Benenati and Brosilow (1962), who determined the voidage variations for ratios of D_c/D_p as low as 2.61. Their results indicate that the voidage variations for all ratios of D_c/D_p are similar, with the exception of $D_c/D_p < 6$ in which a constant value of the porosity (bulk region) is never reached.

Ridgway and Tarbuck (1968) have presented a semi-theoretical prediction which agrees remarkably well with their own data, as well as the data of other investigators. Their work further substantiates the nature of the voidage variations, both near flat and cylindrical walls. Their model was based on considering the voidage fraction for a closed-packed arrangement which was modified by introducing two randomizing parameters. More recently, Kubie (1973) has proposed a simple geometrical function to describe the voidage variations near a flat wall for a distance of up to one particle diameter. Pillai (1977) has verified experimentally the general validity of Kubie's equation, using a two dimensional model to evaluate the porosity variations in a bed made with wooden disks.

MODEL DEVELOPMENT

Modelling of Voidage Variations

It is the purpose of this study to determine the effect of the well-defined variations in porosity with distance from the wall on the pressure drop-flow rate behavior in a cylindrically packed bed.

The analysis is restricted to columns randomly packed with spheres of uniform size for which an adequate quantitative description of the porosity variations have been reported in the literature (Roblee et al., 1958; Ridgway and Tarbuck, 1968; Haughey and Beveridge, 1966). The actual voidage variations for randomly packed beds vary somewhat with the bed to particle diameter ratio, especially for D_c/D_p ratio less than about 10. This variation is mainly due to a phase shift in the porosity oscillations, whose magnitude is at most 0.2 particles diameters, while the actual oscillation amplitude remains about the same. Consequently, rather than treat each bed of a particular D_c/D_p ratio independently, the results of Roblee et al. (1958), and Ridgway and Tarbuck (1968) for packed beds of D_c/D_p range from 7 to 60 have been averaged and taken as a reasonable description of the porosity variations for all packed beds with the above range of D_c/D_p ratio. In this averaging procedure the maximum phase shift is about 0.15 particle diameters, with an average phase shift of about 0.05 particle diameters, while the average deviation in the amplitude of the porosity oscillations was about 5%. This description of the voidage variation, Figure 1, is the basis for the model used in this study.

For the purpose of mathematical convenience, the curve in Figure 1 was fitted by the following set of equations, with an average deviation of 0.5%,

$$\frac{1-\epsilon}{1-\epsilon_b} = 4.5\left(x - \frac{7}{9}x^2\right), \quad x \leq 0.25 \tag{18a}$$

$$\frac{\epsilon-\epsilon_b}{1-\epsilon_b} = a_1 e^{-a_2 x}\cos(a_3 x - a_4)\pi, \quad 0.25 < x < 8 \tag{18b}$$

$$\epsilon(x) = \epsilon_b, \quad 8 \leq x \leq \infty \tag{18c}$$

where x is the distance from the wall nondimensionalized with respect to the particle diameter. ϵ is the local porosity and ϵ_b is the bulk porosity. The constants a_1 through a_4 were determined to be:

$$a_1 = 0.3463; \quad a_2 = 0.4273; \quad a_3 = 2.4509; \quad a_4 = 2.2011 \tag{18d}$$

In order to incorporate the voidage variation into a model for the flow through a packed bed, the packed column has been divided into three regions as shown in Figure 2. The region extending from the wall to x_t is considered to be the wall region. The second region, which extends from x_t to x_b, is defined as the transition region. This region exhibits damped oscillations in the porosity which persist to a distance x_b, beyond which the local porosity is constant and is equal to the bulk porosity. In the subsequent modelling, the wall region was assumed to extend a distance of one particle diameter away from the wall, consistent with the assumption of Rose (1945) and the studies of Schwartz and Smith (1953), Marivoet et al. (1974), and Saunders and Ford (1940). In addition, since the particle diameter is an inner length scale in the bed, it seems reasonable to take the wall region as one particle diameter in thickness. The transition region was arbitrarily assumed to extend to 5 particle diameters away from the wall, Figure 1, since voidage variations beyond 5 particle diameters are less than 5%. Hence, for beds with $D_c/D_p \leq 10$ only the wall and the transition regions exist.

FLOW DESCRIPTION: TRIREGIONAL MODEL

Bulk Region

The porosity in the bulk region is constant (Eq. 18c); hence,

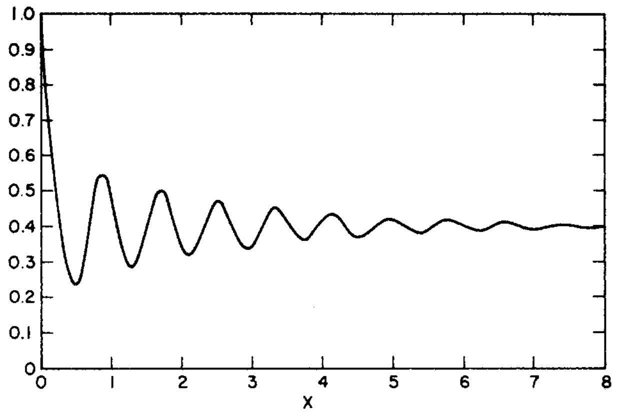

Figure 1. Voidage variations near the wall of a cylindrical column randomly packed with spheres of uniform size.

the definition of the hydraulic radius, Eq. 12 may be substituted into Eq. 10b. The flux in the bulk region can be expressed as:

$$G_b = \rho\left[\frac{\Delta p}{2KL}\right]^{1/n}\left[\frac{4n}{3n+1}\right]\left[\frac{D_p}{3}\right]^{\frac{n+1}{n}}\left[\frac{\epsilon_b}{1-\epsilon_b}\right]^{\frac{n+1}{n}}\left[\frac{\epsilon_b}{2B_b(n)}\right] \quad x > x_b \tag{19}$$

where ϵ_b and $B_b(n)$ are the values of the porosity and the constant $B(n)$ (Eq. 11b) in the bulk region.

Transition Region

The flux, G_t, in the transition region from X_t to X_b can be written as

$$G_t = \frac{1}{A_t}\oint_{A_t} G\,dA \tag{20a}$$

where A_t is the cross sectional area of the transition region and G is the local flux. Substitution of the power law capillary model for G (Eq. 10b), together with the definition of the hydraulic radius (Eq. 12) yields:

$$G_t = \rho\left(\frac{\Delta p}{2KL}\right)^{1/n}\frac{4n}{(3n+1)}\left(\frac{D_p}{3}\right)^{\frac{n+1}{n}}\left\{\frac{1}{A_t}\oint_{A_t}\left(\frac{\epsilon}{1-\epsilon}\right)^{\frac{n+1}{n}}\frac{\epsilon\,dA}{2B_t(n)}\right\}, \quad x_t < x < x_b \tag{20b}$$

where the local porosity is given by Eq. 18.

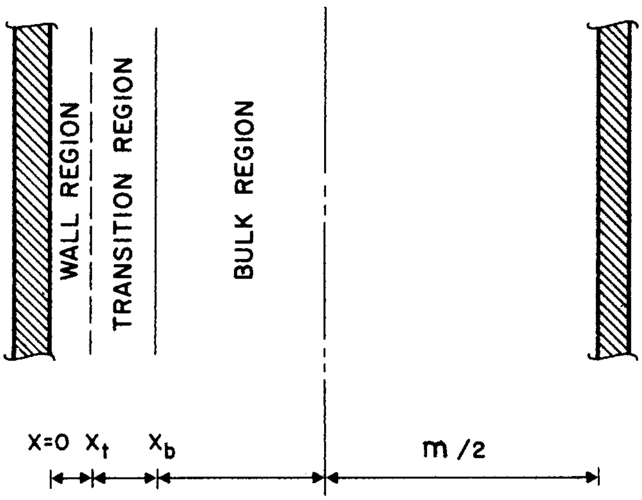

Figure 2. A schematic representation of the triregional model.

Wall Region

In the wall region the usual definition of the hydraulic radius (Eqs. 5 and 12) cannot be used since the presence of the wall is not considered. As the porosity tends to a value of unity (Figure 1), the hydraulic radius (Eq. 12) tends to infinity. The studies of Ergun (1952), Rose (1945), Leva (1947), Carman (1932), MacDonald et al. (1979), and the discussions and references contained in Bird et al. (1960) indicate that the capillary model yields adequate results only for porosities less than about 0.5.

The simplest approach in handling the wall region is to retain the original definition of the hydraulic radius (Eq. 5), and to take the presence of the wall into account in evaluating the wetted surface area as is done in Eq. 13. Recall the definition of the hydraulic radius:

$$R_h = \frac{\text{Volume for flow}}{\text{Wetted surface area}} \tag{5}$$

The volume for flow (or the volume of voids) in the wall region is:

$$\text{Volume for flow} = \left[\left(\frac{D_c}{2}\right)^2 - \left(\frac{D_c}{2} - x_t D_p\right)^2\right]\pi L_c \epsilon_{wav} \tag{21}$$

where D_c is the column diameter, L_c is the length of the column and x_t is the dimensionless boundary of the transition region, Figure 2, and ϵ_{wav} is the average porosity in the wall region, defined as:

$$\epsilon_{wav} = \frac{1}{A_w}\oint_{A_w} \epsilon dA \tag{22}$$

where A_w is the cross sectional area of the wall region. The wetted surface area in the wall region is:

$$A_{\text{wetted}} = \pi D_c L_c + A_{\text{spheres}} \tag{23}$$

where the total surface area of the spheres is:

$$A_{\text{spheres}} = V_{\text{spheres}} x \frac{6}{D_p} \tag{24}$$

and V_{spheres} is the total volume of the spheres in the wall region, determined by:

$$\begin{matrix}\text{Volume of spheres} \\ \text{in wall region}\end{matrix} = \begin{matrix}\text{Total volume} \\ \text{of wall region}\end{matrix} - \begin{matrix}\text{Volume of voids} \\ \text{in wall region}\end{matrix} \tag{25a}$$

$$= \pi\left[\left(\frac{D_c}{2}\right)^2 - \left(\frac{D_c}{2} - x_t D_p\right)^2\right]L_c(1 - \epsilon_{wav}) \tag{25b}$$

The modified expression for the hydraulic radius in the wall region is then obtained by substituting Eqs. 21-25b in Eq. 5, which leads to:

$$R_h = \frac{D_p(m - x_t)x_t\epsilon_{wav}}{m + 6x_t(m - x_t)(1 - \epsilon_{wav})} \tag{26}$$

where $m = D_c/D_p$.

The average mass flux in the wall region G_w can then be determined by using the above expression for the hydraulic radius in Eq. 10b, hence

$$G_w = \rho\left(\frac{4n}{3n + 1}\right)\left(\frac{\Delta p}{2KL_c}\right)\frac{(2R_h)^{\frac{n+1}{n}}}{2B_w(n)}\epsilon_{wav} \tag{27}$$

The apparent mass flux through the packed column G_{at} can be determined from:

$$G_{at} = \frac{G_b A_b + G_t A_t + G_w A_w}{A_c} \tag{28}$$

where A_b, A_t and A_w are the cross sectional areas of the bulk, transition, and wall regions, respectively; A_c is the cross sectional area of the entire column. In order to compare this triregional model to the usual single region model, which is based on the average bed porosity, the ratio of the average mass fluxes for the same packed bed at equivalent pressure drop is expressed (using Eq. 20) as:

$$\frac{G_{at}}{G_{as}} = \frac{G_b A_b}{G_{as}A_c} + \frac{G_t A_t}{G_{as}A_c} + \frac{G_w A_w}{G_{as}A_c} \tag{29}$$

where G_{as} is the flux based on the average porosity as determined by:

$$\epsilon_{av} = \frac{1}{A_c}\oint_{A_c} \epsilon dA \tag{30}$$

where ϵ is the local porosity as determined from Eqs. 18a-18c. G_{as} is then determined by substituting G_{av} in Eq. 19, hence

$$G_{as} = \rho\left[\frac{4n}{3n + 1}\right]\left[\frac{\Delta p}{2KL}\right]^{1/n}\left[\frac{D_p}{3}\right]^{\frac{n+1}{n}} \left[\frac{\epsilon_{av}}{1 - \epsilon_{av}}\right]^{\frac{n+1}{n}}\left[\frac{\epsilon_{av}}{2B_{av}(n)}\right] \tag{31}$$

The various terms in Eq. 29 are then determined by taking the proper ratios as indicated, hence

$$\frac{G_b A_b}{G_{as}A_c} = 4\left[\frac{\epsilon_b}{\epsilon_{av}}\right]^{n+2}\left[\frac{1 - \epsilon_{av}}{1 - \epsilon_b}\right]^{\frac{n+1}{n}} \left(\frac{\frac{m}{2} - x_b}{m}\right)^2 \frac{B_{av}(n)}{B_b(n)} \tag{32}$$

$$\frac{G_t A_t}{G_{as}A_c} = \frac{B_{av}(n)}{\epsilon_{av}}\left[\frac{1 - \epsilon_{av}}{\epsilon_{av}}\right]^{\frac{n+1}{n}} \left\{\frac{1}{A_t}\oint\left[\frac{\epsilon}{1 - \epsilon}\right]^{\frac{n+1}{n}}\frac{\epsilon}{B_t(n)}dA\right\} \left[\frac{\left(\frac{m}{2} - x_t\right)^2 - \left(\frac{m}{2} - x_b\right)^2}{m^2/4}\right] \tag{33}$$

$$\frac{G_w A_w}{G_{as}A_c} = \frac{B_{av}(n)}{B_b(n)}\left[\frac{1 - \epsilon_{av}}{\epsilon_{av}}\right]^{\frac{n+1}{n}} \left\{\frac{G_{wav}}{\epsilon_{av}}\left[\frac{6(m - x_t)x_t\epsilon_{wav}}{m + 6x_t(m - x_t)(1 - \epsilon_{wav})}\right]^{\frac{n+1}{n}}\right\} \left[\frac{\left(\frac{m}{2}\right)^2 - \left(\frac{m}{2} - x_t\right)^2}{m^2/4}\right] \tag{34}$$

It is useful to express Eq. 29 in the following way:

$$G_{at} = \phi_a G_{as} \tag{35}$$

where the correction factor ϕ_a is equal to the R.H.S. of Eq. 29. The correction factor ϕ_a is then equivalent in interpretation to that given by Coulson (1949) and Carman (1938). The relative contribution of the different regions to the apparent flux (Eq. 28) can also be determined by expressing the ratio of the different fluxes to the apparent flux defined in Eq. 28.

COMPUTATIONS

The triregional model was compared to the single region model by evaluating the factor ϕ_a in Eq. 35. This was accomplished by evaluating Eqs. 32 through 35 numerically, using the voidage variations as expressed by Eq. 18. The ratio of the mass flux in the different regions relative to the average flux was also determined by a similar procedure. The calculations were performed at a given pressure drop for beds of varying D_c/D_p ratios.

In order to proceed with the computations, the variations of the parameter $B(n)$ in the three regions must be quantified. Recall the definition of $B(n)$:

$$B(n) = K_o(L_e/L)^{\frac{n+1}{n}} = K_o(C^*)^{\frac{n+1}{n}} \quad (11b)$$

Hitchcock (1926) argued that for a bed packed with spheres $L_e/L = \pi/2$. Carman (1937) believed that L_e/L could not be as large as $\pi/2$. He then conducted flow visualization experiments in a packed bed with $D_c/D_p = 4$. His observation indicated that on the *average* the fluid path line in the first four layers of spheres was at an angle of about 45° with the axis of the column, hence, this led Carman (1937) to assume that $L_e/L = \sqrt{2}$, a value which he adopted for all D_c/D_p ratios. More recently, the flow visualization study of Kubo et al. (1979), in beds packed with spheres in a cubic and rhombohedral arrangement, showed that in the viscous flow regime, the streamlines of a dye tracer showed a creeping flow along the surface of the packed beds. This observation suggests a value of L_e/L close to $\pi/2$ for these highly ordered packings. If we adopt Carman's (1937) value of $L_e/L = \sqrt{2}$ as the lower limit, then we may conclude that the variation in the parameter L_e/L is about 12%; however, if we adopt the capillary model (i.e., $K_o = 2$) then from the range of the reported $B(n = 1)$ values it appears that the variations in L_e/L are about 9.5%. In view of this uncertainty, we also study the effect of L_e/L on the triregional model. However, unless otherwise indicated, the discussion and figures refer to the results with a uniform L_e parameter.

DISCUSSION

In order to put the calculated results in the proper perspective, the contribution of the areas of the different regions for different D_c/D_p ratios is shown in Figure 3. The transition region is the largest fraction of the total area below D_c/D_p of about 30 and the bulk region makes the largest contribution above $D_c/D_p = 30$. The wall region is the smallest fraction at $D_c/D_p > 18$, although below $D_c/D_p = 18$ its area is actually larger than the bulk area. This division of the relative areas is important to the understanding of the effect of each region on the average mass flux through the packed bed.

The flux in the bulk region is found to be lower than the average flux, Figure 4. This is most easily visualized by the fact that the effective pore diameter in the bulk is smaller than the average pore diameter by as much as 8% at the low D_c/D_p ratios.

The flux in the transition region is appreciably higher than the average flux, Figure 5. The overall effect of the voidage variations in this region is to yield the equivalent of a higher porosity than in the bulk or the wall region. This leads to a higher degree of channeling through this region and this is especially apparent as the power law index decreases. The wall region (Figure 6) presents an interesting result. While the average porosity in the wall region is greater than the bulk porosity by about 15%, the effective pore diameter is essentially identical to the pore diameter in the bulk. The net result is a higher flux in the wall region as compared to the bulk region, but yet smaller than the flux in the transition region. This would appear to be of great importance in modelling heat transfer rates and gas-liquid contacting in packed columns, for example. In both the wall and bulk regions, the ratio of the individual fluxes to the average flux decreases with decreasing power law index, Figures 4 and 6. This trend is due to the more rapid channeling through the transition region, Figure 5, as the power law index decreases. This leads to a higher overall or average flux with a decreasing power law index.

In summary, it appears that the flux is greatest in the transition region, lower in the wall region, and lowest in the bulk region. The total effect of the above on the average flux depends, of course, on the area fraction of each of the regions, Figure 3. Since the porosity in the bulk region is independent of D_c/D_p (Figure 1), the mass flux in the bulk region (Eq. 19) is also independent of D_c/D_p. Consequently, we can conclude from Figure 4 that the average flux decreases as the column to particle diameter ratio increases, its magnitude being larger as the power law index decreases.

The comparison of the triregional model to the single region model based on the average bed porosity, is presented in Figure 7. The comparison is made at the same pressure drop for differ-

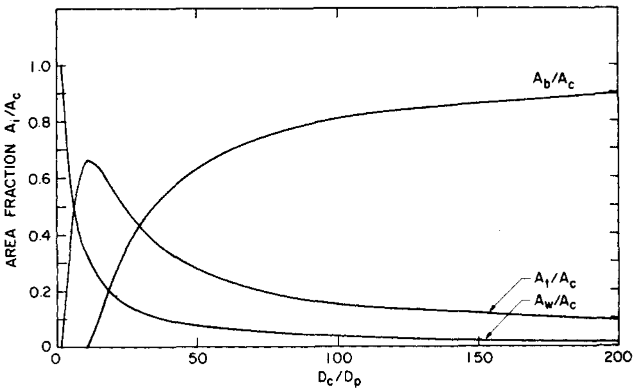

Figure 3. Area fractions of the different regions in the triregional model for a cylindrical column.

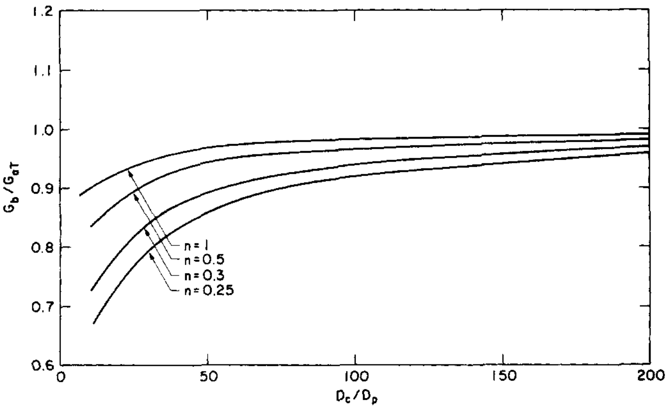

Figure 4. The ratio of the bulk mass flux to the average mass flux vs. D_c/D_p for different power law indices (triregional model).

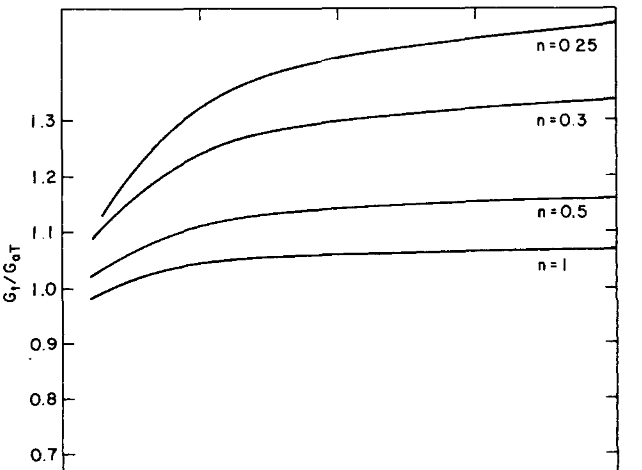

Figure 5. The ratio of the mass flux in the transition region to the average mass flux vs. D_c/D_p for different power law indices (triregional model).

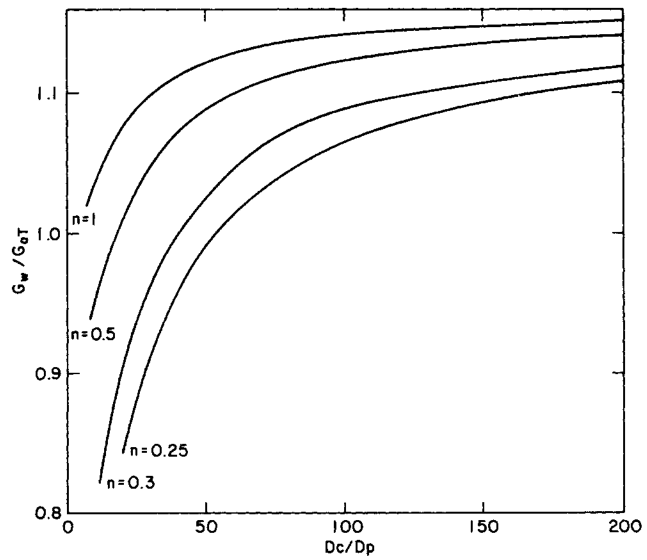

Figure 6. The ratio of the mass flux in the wall region to the average mass flux vs. D_c/D_p for different power law indices (triregional model).

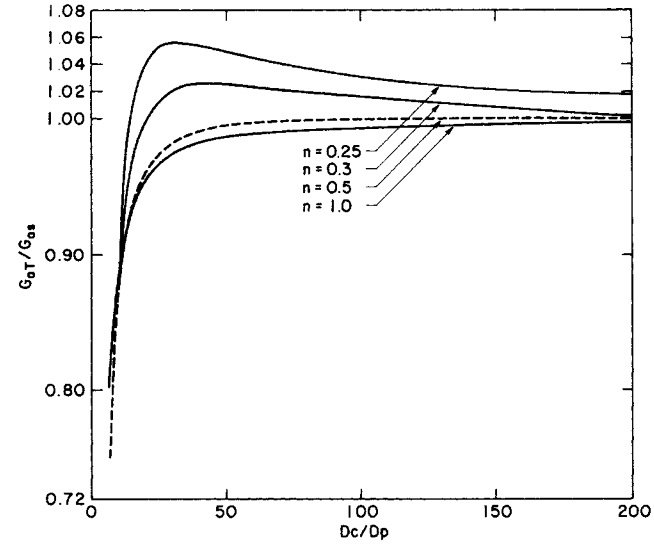

Figure 7. A comparison between the average mass flux predicted by the triregional and single region models vs. D_c/D_p for a uniform L_e and different power law indices.

ent power law indices. For Newtonian fluids (i.e., $n = 1$) the flow rate predicted by the triregional model is lower than the single region model prediction. The error in using the single region model is 10% at D_c/D_p ratio of 12 and increases dramatically as the bed to particle diameter ratio decreases further. From a practical point of view, it appears that for Newtonian fluids, the use of a bed to particle diameter ratio greater than about 30 will lead to errors which are no more than 3%, and hence is adequate for most experimental studies.

For power law fluids, for a given pressure drop, the interstitial velocity depends on the hydraulic radius, i.e.,

$$V \propto (R_h)^{\frac{n+1}{n}}$$

The exponent on R_h is larger for non-Newtonian fluids for which $n < 1$, thus shear thinning fluids exhibit a greater tendency to channel through the most permeable parts of the porous media. Consequently, the increase in the mass flux in the wall and transition regions is greater than for Newtonian fluids. As is shown in Figure 7, this trend increases as the power law index decreases and eventually leads to an underestimation of the average flux, if one uses the average bed porosity. For power law indices greater than 0.25, the minimum value of D_c/D_p recommended for Newtonian fluids ($D_c/D_p = 30$) leads to errors less than 6% in calculations based on the average bed porosity, but D_c/D_p must exceed 100 for the error to drop below 3%. It should be noted that when the bed porosity is determined from Eq. 10b via calibration experiments with Newtonian fluids, the resulting porosity is essentially an "effective porosity." The use of such an "effective porosity" will probably improve the prediction of the Ergun equation for Newtonian fluids through the same bed. However, the use of such effective average porosity in Eq. 10b for power law fluids ($n < 1$) especially for beds with small D_c/D_p, ratios may result in appreciable errors.

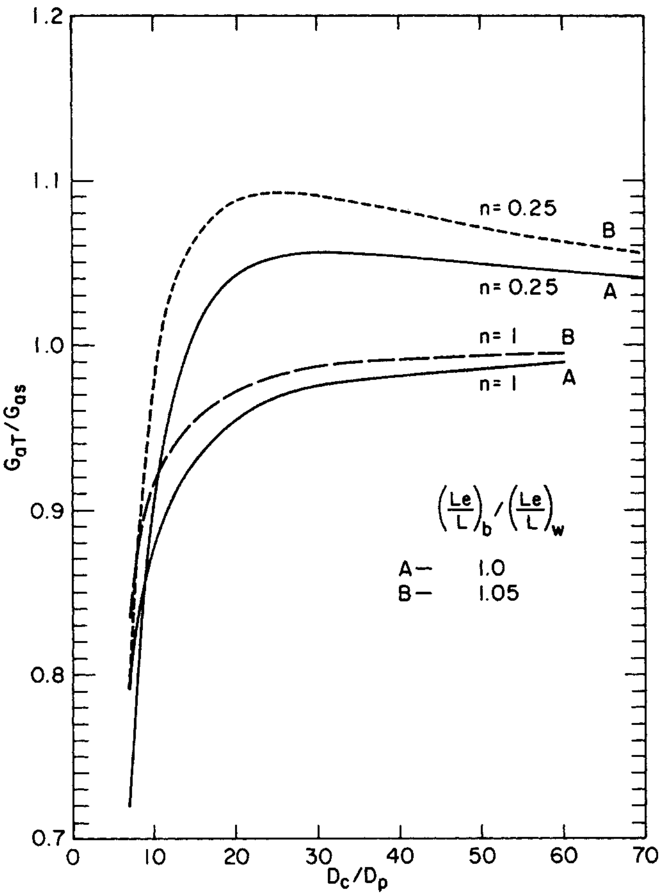

Figure 8. A comparison between the average mass flux predicted by the triregional and single region models vs. D_c/D_p for a variable L_e and different power law indices.

Radial Variations of the L_e Parameter

It appears reasonable to assume that near the wall the effective path length is less tortuous than in the bulk. This is consistent with the observation of Carman (1937), in flow visualization experiments with a packed bed of D_c/D_p ratio of 4. It may also be reasonable to assume that L_e/L in the bulk and transition regions is identical. Furthermore, the capillary model is accepted ($K_o = 2$), thus allowing only L_e/L to vary in this analysis. If, for example, it is assumed that:

$$C^*_{\text{bulk}}/C^*_{\text{wall}} = (L_e/L)_{\text{bulk}}/(L_e/L)_{\text{wall}} = 1.05$$

then, the triregional model will lead to a somewhat higher mass flux prediction (for a given Δp) for both Newtonian and power law fluids (Figure 8), as compared to the case of a uniform Le across the bed. Computations with larger $C^*_{\text{bulk}}/C^*_{\text{wall}}$ ratios revealed, as may be expected, that as $(L_e/L)_{\text{wall}} \to 1$, a larger degree of channeling at the wall region results, especially for highly shear thinning fluids. In the next section, available experimental results are used in an attempt to establish the importance of such variations in L_e.

COMPARISON WITH EXPERIMENTAL DATA

Newtonian Fluids

The data of several investigators for the flow of Newtonian fluids (listed in Figure 9) were employed in order to test the applicability of the triregional model. A particular data set was employed only if the bed porosity and particle diameter were determined by measurements other than flow calibration, in order to ensure that the two parameters were indeed independent. In addition, the data were selected to satisfy, as nearly as possible, the requirement of uniform particle size and random packing, in accordance with the present analysis. The parameter $B(n)$ in Eq. 31 was determined for each bed with a given D_c/D_p ratio by a least squares fit of the data, using Eq. 31 and the reported average bed porosity and average particle diameter.

In order to compare the present results to both experiments and other proposed empirical and experimental correction curves (see Eq. 2), it is instructive to compare the ratio of the mass flux for a finite size bed to that of an infinite bed for a given pressure drop ($\Delta p/L$), all other factors being constant, based on Eq. 31. This is equivalent to comparing the ratios of $B(n = 1)$ for an infinite bed diameter to that of a finite bed. In our analysis of the available experimental data, an infinite bed was designated to be one with a ratio greater than about 30, due to the lack of data with very large D_c/D_p ratios. For studies where only D_c/D_p ratios less than 30 were employed, it was decided to test the recent growing support for a value of $B(n = 1) = 5$ and use it as representative of an infinite bed value. The theoretical values of the ratio of the mass flux for a finite bed to that of an infinite bed were based on Eq. 29; those are equivalent to the correction factor defined in Eq. 35 or equivalently the ratio

$$B(n = 1)_{\text{infinite bed}}/B(n = 1)_{\text{finite bed}}.$$

The comparison for Newtonian fluids is shown in Figure 9. The triregional model is in good agreement with the empirical correction of Coulson (1949) and agrees approximately with the empirical curve obtained by Rose (1948). The data of Mehta and Hawley (1969) seem to exhibit a greater deviation for $D_c/D_p \leq 30$ than expected. It appears that the number of data points in Mehta and Hawley's (1969) work in the laminar (viscous) regime are too few and the scatter of the data is too large for the determination of the magnitude of the wall effect with a high level of confidence. The correction that Mehta and Hawley (1969) employ (Eq. 14) does not yield better agreement with the data than the triregional model. Nevertheless, the results are still in qualitative agreement with the general trend depicted in Figure 9. The data of Sadowski (1963), Park et al. (1975), Elata et al. (1977), and Mishara et al. (1975) were plotted (Figure 9) based on a value $B(n = 1) = 5$ and are in reasonable agreement with the predicted curve, if not with the general trend. The results do indicate that above D_c/D_p of about 30, the single region model is adequate since the wall effect is minimal.

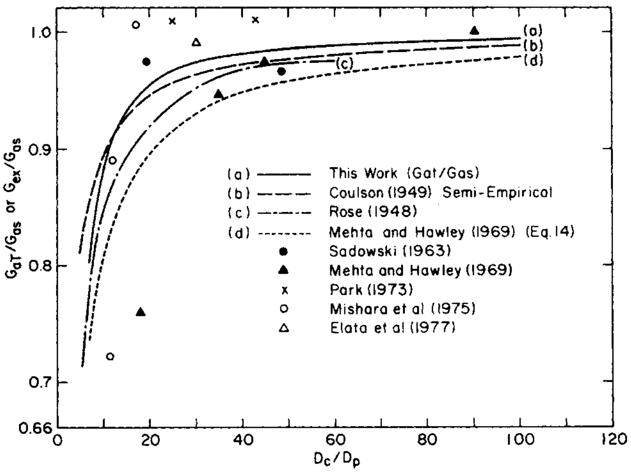

Figure 9. A comparison between experiment and predictions of the average mass flux (relative to the single region model prediction) vs. D_c/D_p for Newtonian fluids.

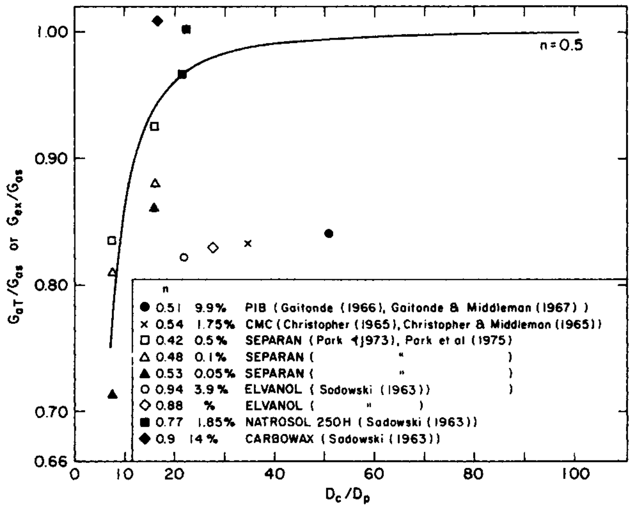

Figure 10. A comparison between experimental average mass flux and the triregional model prediction relative to the single region model prediction vs. D_c/D_p for non-Newtonian fluids.

Non-Linear Fluids

Studies on the flow of non-linear fluids through packed beds generally do not cover a wide range of D_c/D_p ratios for experiments with a given fluid and hence do not allow for an adequate estimate of $B(n)$ for an infinite bed. Therefore, we selected to employ the values of $K_o = 2$ (appropriate for the capillary model) and a value of $\sqrt{2.5}$ for L_e/L based on $B(n = 1) = 5$, a value which appears to represent most of the Newtonian data at large D_c/D_p ratios (see previous discussion and Kemblowski and Michniewicz, 1979). The ratios of the experimental mass fluxes to the fluxes predicted from Eq. 31 for a given pressure drop were calculated and averaged for each given bed and fluid pair, and the results plotted against the bed to particle diameter ratio.

The comparison for power law fluids having power law indices in the range of 0.42 to 0.94 is illustrated in Figure 10, together with the predicted curve for $n = 0.5$. The single region model overpredicts the experimental mass fluxes (for the given data) as the bed to particle diameter ratio decreases (Figure 10). The deviation of the data from the single region model is closely predicted by the triregional model; in other words, the triregional model describes the experimental data much more closely for small D_c/D_p ratios. Exceptions are the concentrated Elvanol solutions of Sadowski (1963), CMC solution of Christopher and Middleman (1965), and PIB solution of Gaitonde and Middleman (1967), which all fall below the prediction of both the single and triregional models by about 18%.

A problem which remains partially unresolved is a description of the exact nature of the radial variations of L_e in the packed bed. We note that the approximate agreement of the triregional model (for $0.42 \leq n \leq 1$), with a uniform L_e, with the available experimental data (Figures 9 and 10) indicates that while L_e may vary somewhat, the main effect on the flow behavior in the bed is probably due to the porosity variations and the wall presence. An accurate description of L_e may be desired, for example, in assessing the wall effect on heat transfer in tubular reactors.

TABLE 1. D_c/D_p RATIO.

Investigator	D_c/D_p
Sadowski (1963, 1965)	14.2-485
† Christopher and Middleman (1965)	30.3-35.8
† Gaitonde and Middleman (1967)	30.3-35.8
Dauben and Menzie (1967)	71-500
Gregory and Griskey (1967)	20-46.9
Yu et al. (1968)	11.4-50
James and McLaren (1975)	58-236
Brea et al. (1976)	16.4-46.2
Mishara et al. (1976)	9.71-16.85
Laufer et al. (1976)	16
Mena et al. (1976)	4.25-58.1
* Michele (1977)	5-100
* Hanna et al. (1977)	16.8
* Park et al. (1975)	7.8-42.7
Parker (1977)	10.52-88
Naudasher and Killen (1977)	131-278
Elata et al. (1977)	30-60
Kemblowski and Dziubinski (1978)	8.66-10.61
** Wang et al. (1979)	10-573

* Used Eq. 14 to correct for wall effect.
** Cemented a unilayer of spheres to the pipe wall.
† Porosity or particle diameter determined from flow calibration.

However, such information can only be obtained through careful experiments with beds of small D_c/D_p ratio and/or accurate velocity profile measurements, coupled with local porosity measurements.

Finally, we note that there are numerous studies dealing with non-Newtonian fluids, some of which have failed to consider the wall effect or have attempted to account for it, based on the correction factor suggested by Mehta and Hawley (1969) for Newtonian fluids (see Eqs. 14 and 15). These have been compiled and are listed in Table 1.

In conclusion, it is emphasized that in scaling laboratory experiments it is best to use beds with large D_c/D_p ratio. Data from beds with small D_c/D_p ratio can still be used for crude estimates if proper corrections are made. In scaling pressure drop-flow rate experiments one can use any one of the correction methods described in this work (Eqs. 2-3, 16-17, 29 and 35). However, in scaling experiments where heat and mass transfer or residence time considerations are important one should consider the detailed description of the voidage variations as described by the triregional model.

NOTATION

A_c = cross-sectional area of packed column
A_b, A_w, A_t = cross-sectional area of the bulk, wall and transition regions, respectively
a_1, a_2, a_3, a_4 = constants defined in Eq. 18d
$B(n)$ = parameter defined in Eq. 11b
$B_b(n)$, $B_t(n)$, $B_w(n)$ = value of $B(n)$ in the bulk, transition, and wall regions, respectively
$B_{av}(n)$ = average value of $B(n)$ for the packed bed
C^* = parameter defined in Eq. 10c
C', C'' = constants in Eq. 3
D_c, D_p = diameters of packed column and particle, respectively
D_e = effective channel radius (see Eq. 4)
f = friction factor defined by Park et al. (1973)
G = mass flux
G_{at} = average mass flux through the column as calculated from the triregional model
G_{as} = average mass flux through the column calculated from the single region model
G_b = mass flux in the bulk region
G_{ex} = experimental mass flux
G_w = mass flux in the wall region
G_t = mass flux in the transition region
k = permeability (Eq. 7)
K = rheological power law parameter (consistency index)
L = length of porous medium under consideration
L_c = length of packed bed
L_e = effective path length
M, M' = parameters as defined in Eqs. 15 and 17, respectively
m = ratio of bed to particle diameter
n = rheological power law parameter
p = pressure
Δp = pressure drop
Q_n = flow rate normal to the column cross-sectional area
Re = Reynolds number
R_h = hydraulic radius (see Eq. 5)
S_o = packing surface area per unit volume of empty column
S_w = surface area per unit volume of column
t = time
V = interstitial velocity
V_n = interstitial velocity component parallel to the bed axis
V_o = superficial velocity
$V_{spheres}$ = volume of spheres
x = distance from the wall nondimensionalized with respect to D_p
x_b, x_t = boundary of the bulk and transition regions (dimensionless distance)

Greek Letters

ϵ = local porosity
ϵ_{av} = average bed porosity
ϵ_{wav} = average porosity in the wall region
ρ = fluid density
ϕ_o = correction term defined in Eqs. 2, 3a, 3b, 35
μ = Newtonian viscosity
μ_a = apparent viscosity

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